**Special Relativity Postulates**

Let’s continue our analysis of the physical consequences of the relativity postules, which are, we’ll recall.



We’ll consider length contraction now.

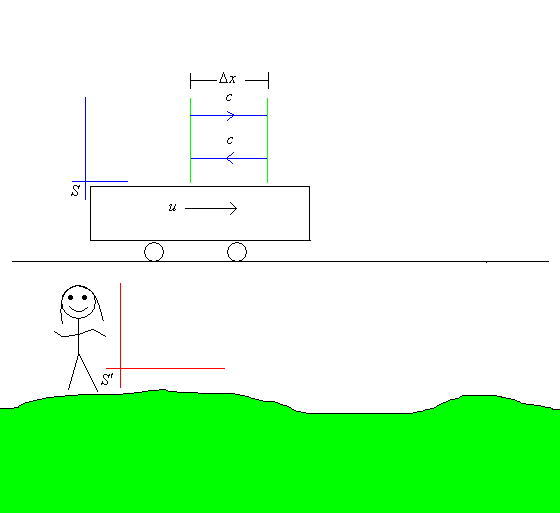
**Spatial distance between two events transformation**

Suppose two events occur simultaneously in a reference frame (meaning they would be inferred to have occurred at the same time by an observer in that frame). Then this reference frame is called the proper reference frame, S­­­­, with respect to the distance between the two events. Another way to consider it…suppose you have a length, Δx, at rest in your reference frame (meaning the length’s coordinates do not change in your reference frame), then Δx is considered its proper length. And to relate this to our previous definition, we can say that if, according to the observer in that frame, two light signals are emitted at the same time, then the proper length between those two events is Δx.

So in any event, the distance between the two events measured in this frame is denoted Δx. The distance measured in another inertial frame of reference, S′, is denoted Δx′. We can show that the relation between the two is:



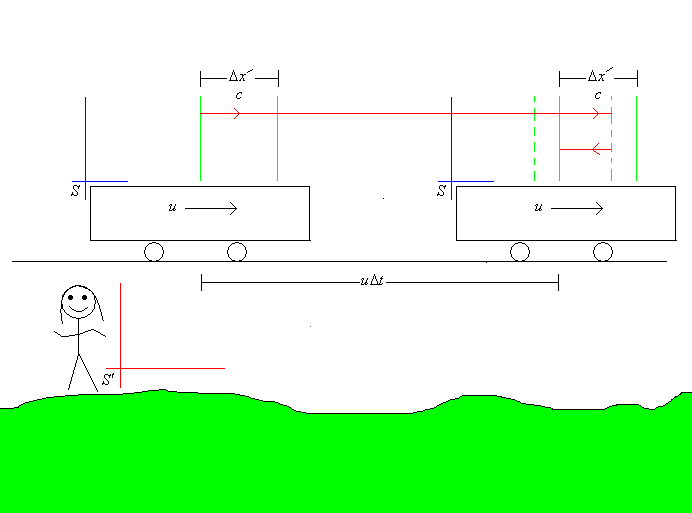
which states that a distance measured in a moving frame of reference is shorter than the distance measured in the proper frame – the opposite of the relationship between times. We can derive this relation. Consider the moving train again, only this time, flip our mirror, and call the distance between the mirrors, in the reference frame of the train, Δx.



In the frame of reference of the train, the light travels back and forth a distance 2Δx, with velocity c, in a time Δt. And we have,



Now consider these events from the frame of reference of the bystander,



The whole process takes a time Δt′ = γΔt. The distance traveled is tricky. We have to determine how long the light ray is going forward, and how long the light ray is going backward separately, to determine the total distance traveled (it isn’t exactly what the illustration might have you believe). We can work all these out in terms of the known speed of light, c, and the perceived length of the mirror, Δxʹ. Let Δt′*f* be the time going forward, and let d′*f* be the distance it travels going forward. Then we have,



Solving for Δt′f, and plugging into the other we get,



and for the backward motion we have,



Solving for Δt′b and plugging into the other we have,



Therefore the total distance traveled is:



and this is equal to the light wave’s velocity × time,



and now use the relation from the proper frame,



solve for Δt and plug into the Δx′ equation,



which comes to:



An alternate, I think simpler, derivation would be. Figure out the time it takes for the light ray to hit the front end of the mirror. This is:



Then the time required to hit the back end would be:



Then we can say, adding the times together:



and we know that Δt΄ = γΔt so:



and we know that 2Δx = cΔt. So then,



Note that length will appear to be contracted in both reference frames (similar to how – Wikipedia analogy – people appear small to each other at a distance in both reference frames.

**Example: Fitting car in garage**

Suppose have really fast car, 2.5m long, and a garage that is only 2m long. How fast do you have to move the car in order to fit the car into the garage?

Well, when the car is at rest, its length is 2.5m. This is its proper length. When moving at velocity υ, we will measure its length to contract to Δx′ = Δx/γ, which we need to equal 2m. So solving for u,



and so we need,



What will happen as we slow the car down to rest? The car will expand to its rest length of 2.5m and punch through the garage – unfortunately.

**Example: Particle decay**

For instance, suppose that we have a hadron hurtling into the Earth’s atmosphere at 0.9c. Its lifetime in its frame of reference is say 5μs. How long will it appear to last in our frame? Well, the proper time is Δt = 5μs. The amount of time it will last in our frame is:



How far will it travel? Well, in our rest frame it will travel:



And this is the proper reference frame since we are rest when the measurement is made. Or in other words, the positions of the beginning and end of the event are not changing in our reference frame. How far will it travel in its reference frame? Well, the length will appear to be contracted to it. It will think it goes as:

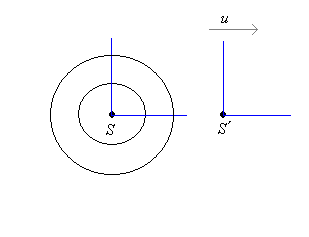


We can also calculate the distance from its perspective, it would be:



**Relativistic Doppler effect**

Suppose a source S is emitting waves with a frequency *f* and an observer is moving with speed u w/r to the source. What is the frequency of waves intercepted by the observer S´?



It’s easiest to answer this question by examining the wavelengths. According to S, the wavepulses are emitted every Ts seconds. Morever, the first wavepulse will have traveled a distance λs = cTs away by the time the second wavepulse gets emitted. But according to the observer, these pulses are emitted every To = γTs seconds. And the first wavepulse will have traveled, to the right say, a distance cTo, and the source to the left a distance uTo. And so the distance between the crests will be (c + u)To. Both S and S´ will agree, however, on the speed of the signal, c. So, according to the observer, the frequency of the waves will be:



And so we have:



[Note that for this formula, u is the speed of the observer relative to the source! In Physics 3 notes u is defined as speed of source relative to the observer and so u -> -u!] Note this analysis applies to the half of the wave that the observer actually intercepts (the right half), not the other half. Also, we did the analysis assuming the observer is moving away from the waves. If the observer were moving towards the waves it is intercepting, then we should switch the sign of u. Another way we could do this is the following. Using simple kinematics, we can calculate the length of time between pulses as they catch up with the observer. So according to the observer, the position of the first and second pulse, and observer as a function of time, will be:



where -x0 is the initial position of the source w/r to the observer, and -x0 – uγTs is the position of the source w/r to the observer by the time the second pulse is emitted. Then the time it takes for the two pulses to catch up to the observer is found by settting x1 and x2 = x­o respectively. We get:



The difference in time is To,



Inverting, we get:



So clearly, light is shifted towards a higher frequency if the source is moving toward you, and to a lower frequency if the source is moving away you – just like the usual Doppler effect, at least qualitatively. Hubble’s (?) observation that the emission spectra of stars were all red shifted implied that all galaxies are moving away from us – which leads to the conclusion that the galaxy is expanding.

**Doppler Broadening**

Consider a star rotating about its axis with equatorial velocity ve (the speed of a point on the equator). The spectral lines emitted from the star with have some rest wavelength λs. But due to the star rotating, some of the atoms emitting these lines will be moving away from us and some will be moving towards us. If the axis of rotation of the star is in the plane of the sky, then the maximum speeds towards and away from us will be ±ve. Let’s work out by how much this will broaden a spectral line of wavelength λs. We’ll just use the Doppler shift formula for speeds small in comparison to light. If we take the small u limit, we have:



We can write this as:



So finally,



So the relative shift will be ve/c for the points on the star’s equator moving towards us, and -ve/c for points moving away. So the total broadening will be:



What if the star’s axis of rotation is not in the plane of the sky? Let θ be the angle of inclination of the axis of rotation w/r to the sky. Then we can work out that,



**Relative Doppler Law?**

Consider situation where we have a source, and two observers. One moving with velocity vo1 while another moves with velocity vo2. We’ll pretend that observer 2 is to the right of observer 1. Then we know that,



Question is, is there a relationship between fo2 and fo1 directly, say like:



Let’s see,



This is indeed the relationship between fo2 and fo1, as we can see directly from the first two formulas above. So there really isn’t anything special about the source. Contrast this with the Doppler Effect formula,



If we had two observers, like above then:



which isn’t the Doppler Effect formula directly relating o2 and s. This I’ll take to be indicative of there being a special rest frame for sound, but not for EM waves.